Technical Notes

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Laser Velocimeter and Hot-Wire Anemometer Comparison in a Supersonic Boundary Layer

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Nomenclature

a_w = wire overheat = $(R_{wire} - R_{ref})/R_{ref}$
E = bridge voltage
M = Mach number
p = pressure
$R_{(\rho u)(T_t)}$ = correlation coefficient for mass flux and total temperature
R_{uv} = correlation coefficient for u and v velocity components
T = temperature
u = velocity component in streamwise direction
u_τ = friction velocity = $(\tau_w/\rho_w)^{1/2}$
v = velocity component in cross-stream direction
y = cross-stream coordinate measured from tunnel wall
Δe = fluctuation sensitivity
ρ = density
σ = standard deviation
τ = shear stress
$\langle\ \rangle$ = standard deviation (rms) of quantity

Subscripts
t = total conditions
w = wall
∞ = freestream conditions

Superscripts
$(\ )'$ = fluctuating quantity
$(\bar{\ })$ = time-averaged quantity

I. Introduction

UNTIL recently, hot-wire anemometry represented the only measurement technique with the potential of measuring velocity fluctuations in supersonic flows. With the emergence of laser velocimetry, an independent means of measuring velocity fluctuations is available. However, certain questions arise as to the accuracy that can be obtained in compressible flows with either measurement technique. For the hot-wire anemometer, calibration procedures and the signal interpretation used to resolve the fluctuations of the physical variables (velocity, density, total temperature, pressure, etc.) become quite complex and are subject to error. The laser velocimeter, on the other hand, is solely velocity-sensitive (especially important is its insensitivity to pressure fluctuations), which makes signal interpretation unambiguous. Unfortunately, it is susceptible to measurement errors caused by particle lag.

To test the validity and accuracy of the laser velocimeter and the hot-wire anemometer for measuring turbulence properties in compressible flows, a study involving both measurement systems was undertaken in the Ames 20.32- by 20.32-cm Supersonic Blowdown Tunnel. Both devices were used independently to measure the turbulence intensities, $\langle u'\rangle$ and $\langle v'\rangle$, and their correlation, $\overline{u'v'}$, across the nearly adiabatic, zero pressure gradient, tunnel-wall boundary layer. Test conditions were a freestream Mach number of 2.9 and a stagnation pressure and temperature of 6.8 atm and 291 K, respectively. All measurements were obtained at a single streamwise station at which the boundary-layer thickness was approximately 2.5 cm.

The mean-flow information (density profiles, Mach number profiles, etc.) used here was obtained in the study of Ref. 1. Laser velocimeter measurements of this flow were reported previously.[2] However, in those measurements, the turbulence quantities at each measurement station were deduced from a very small number of instantaneous velocity readings (several hundred). As a result, the statistical uncertainty associated with these small samples was quite large, causing considerably scatter in the $\overline{u'v'}$ data and making the determination of $\langle v'\rangle$ from the data impossible.

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II. Measurement Techniques

Laser Velocimeter

The laser velocimeter system described in Ref. 2 was used in the present investigation. Since the system has only a one component sensitivity, the realization of $\overline{u'v'}$ and $\langle v'\rangle$ requires signal differencing of measurements obtained at three laser beam orientations. In particular, the measurements are made with the velocimeter sensing u, $1/(2)^{1/2}(u+v)$, and $1/(2)^{1/2}(u-v)$ from which the desired quantities are resolved in a manner analogous to the "slanted hot-wire" technique for constant property flows. As in Ref. 2, instantaneous velocity readings were obtained from the single-particle bursts produced by the naturally occurring particles in the flow with a counter-type processor. The mean values and standard deviations (rms) of these readings were then estimated using standard statistical methods. To remove spurious readings caused by infrequent noisy signals, a 3σ test[2] was applied to the data. The occurrence of these spurious readings was sufficiently rare that only in the region of low turbulence (i.e., the outer third of the boundary layer) was their effect on the data noticeable.

The $\langle u'\rangle$ results, obtained directly by sensing u only, were nominally based on 1000 instantaneous velocity readings at each measurement station. However, to reduce the data scatter in $\overline{u'v'}$ caused by differencing two relatively large numbers, additional samples were taken at the other two orientations. In the inner third of the boundary layer, 6000 velocity readings, nominally, were taken at these orientations and 3000 velocity readings, nominally, in the remainder of the boundary layer.

In laser velocimetry, it is crucial that the light-scattering particles follow the fluid motion. The frequency response of the naturally occurring particles in the subject facility has been determined from laser measurements made across an oblique shock wave. The relaxation distance for the particles downstream of the shock wave was approximately 0.4 cm, which corresponds to a frequency response of 24 kHz in the moving frame of reference of the particles. In a fixed or Eulerian frame of reference, for which frequency responses are usually given, this frequency would correspond to a frequency several fold greater because of the large convection velocities in a supersonic boundary layer.[2]

Hot-Wire Anemometer

To outline the techniques used here to interpret the hot-wire signals, a brief discussion of the response of a hot wire in terms

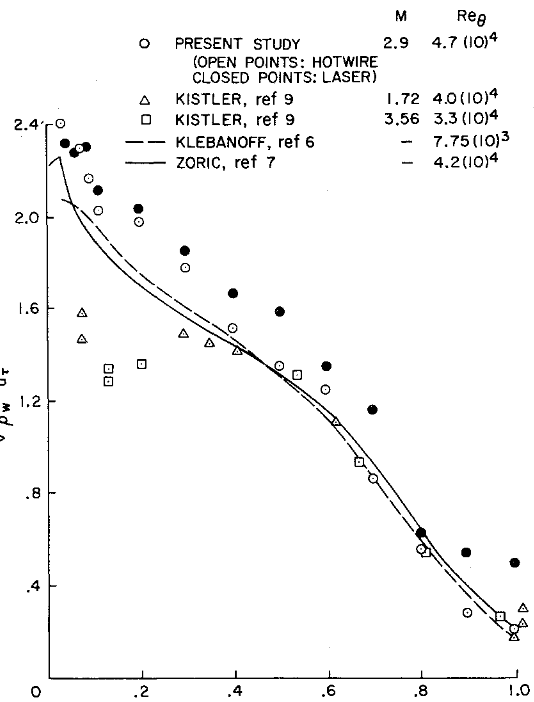

Fig. 1 Streamwise velocity fluctuation.

of its voltage fluctuation to the sensed physical fluctuations is presented. The basic voltage equation applicable to a (possibly yawed) wire in supersonic flow can be written:

$$E' = \Delta e_{\rho u}[(\rho u)'/\bar{\rho}\bar{u})] + \Delta e_{T_t}(T_t'/\bar{T}_t) \pm \Delta e_v(v'/\bar{u}) \qquad (1)$$

where $\Delta e_{(\ )}$ denotes the sensitivity coefficients for each physical variable. These coefficients were determined in this study by direct calibration as described in Ref. 3.

For a normal wire ($\Delta e_v = 0$), the variables $\langle(\rho u)'\rangle$, $\langle T_t'\rangle$, and $R_{(\rho u)(T_t)}$ can be determined from the mean square of Eq. (1) by operating the wire at different temperatures.[3] To obtain $\langle u'\rangle$ from the resulting values of $\langle(\rho u)'\rangle$, $\langle T_t'\rangle$, and $R_{(\rho u)(T_t)}$, the pressure fluctuations p' were assumed to be zero for all locations across the boundary layer. No direct technique for assessing the accuracy of this assumption is available; however, comparisons of $\langle u'\rangle$ obtained using the $p' = 0$ assumption can be made with the $\langle u'\rangle$ values from the laser velocimeter, which do not rely on any pressure fluctuation assumptions.

The cross-stream velocity fluctuation $\langle v'\rangle$ may be found directly through the use of Eq. (1) (not necessarily neglecting p') by instantaneously differencing and then time-averaging the signals from two matched yawed wires held at plus and minus angles to the flow to give $\langle E'\rangle = 2\Delta e_v(\langle v'\rangle/\bar{u})$. The values of $\langle v'\rangle$ in this study were obtained from this technique. Since only one overheat ratio is required to extract $\langle v'\rangle$, the ratio with the best matched Δe_v can be chosen. A mismatch of 5% in Δe_v, which was considered acceptable in the present study, contributes about a 7% error in $\langle v'\rangle$.

The determination of the correlation $\overline{u'v'}$ is considerably more complex than for $\langle u'\rangle$ and $\langle v'\rangle$. The technique used here and in Refs. 3 and 4 is based on the square of Eq. (1) using two individually calibrated wires at $\pm 45°$ to the stream direction:

$$\overline{E'^2} = (\Delta e_{\rho u})^2 \frac{\overline{(\rho u)'^2}}{(\bar{\rho}\bar{u})^2} + (\Delta e_{T_t})^2 \frac{\overline{T_t'^2}}{\bar{T}_t^2} + (\Delta e_v)^2 \frac{\overline{v'^2}}{\bar{u}^2} + 2\Delta e_{\rho u}\Delta e_{T_t} \times \frac{\overline{(\rho u)'T_t'}}{\bar{\rho}\bar{u}\bar{T}_t} \pm 2\Delta e_{T_t}\Delta e_v \frac{\overline{T_t'v'}}{\bar{T}_t\bar{u}} \pm 2\Delta e_{\rho u}\Delta e_v \frac{\overline{(\rho u)'v'}}{\bar{\rho}\bar{u}\bar{u}}$$

Again, data obtained at different wire temperatures can be used[3] to obtain $\overline{T_t'v'}/\bar{T}_t\bar{u}$ and $\overline{(\rho u)'v'}/\overline{\rho u}^2$ and again, for $p' = 0$, to obtain $\overline{u'v'}$.

The anemometer systems used here were DISA 55D01, constant-temperature anemometers. The details of anemometer operation are essentially the same as those described in Ref. 3. The probe wires, 5-μm unplated tungsten, were welded to the probe supports. The resulting lengths were about 0.9 mm for the normal wires and 1.3 mm for the yawed wires. The overheat ratios used here were $a_w = 0.2, 0.3, 0.4, 0.6$, and 0.8. The upper frequency response was above 100 kHz for all overheat ratios as determined by the linear system square-wave test.

III. Results

The turbulent intensities $\langle u'\rangle$ and $\langle v'\rangle$ as determined from the laser velocimeter and the hot-wire anemometer are shown in Figs. 1 and 2, respectively. They are plotted in the dimensionless form suggested by Morkovin[5] to incorporate compressibility effects. As indicated in these figures, the agreement between the independently obtained measurements is quite good; the $\langle u'\rangle$ results compare more favorably than the $\langle v'\rangle$ results. As previously discussed, the realization of $\langle u'\rangle$ from the hotwire signal assumes that the pressure fluctuations can be neglected, whereas no such assumption is necessary in determining $\langle v'\rangle$. With this in mind, a better correspondence might be expected between the hot-wire and laser velocimeter results in $\langle v'\rangle$ than in $\langle u'\rangle$. However, the measurement of $\langle v'\rangle$ is not nearly as direct as that for $\langle u'\rangle$ with either measurement technique. The necessity of taking measurements at more than one laser beam orientation and the use of a two-wire probe contribute to additional experimental inaccuracies. The good agreement in the $\langle u'\rangle$ measurements, however, strongly supports the validity of assuming negligible contributions from pressure fluctuations, at least in the present attached, adiabatic, supersonic boundary layer.

Included in Figs. 1 and 2 for comparison are the incompressible results of Klebanoff,[6] Zoric,[7] and Tieleman,[8] and the compressible results of Kistler[9] for $M = 1.72$ and 3.56. Figure 1 shows that the present $\langle u'\rangle$ data compare favorably with the incompressible measurements. However, there are significant differences between the present $\langle u'\rangle$ data and that of Kistler near the wall. These differences do not appear attributable to effects of either Mach number or Reynolds number. No conclusions are drawn from the comparison of $\langle v'\rangle$ with the incompressible results since the incompressible data are inconsistent by themselves. The present data, however, agree more favorably with the

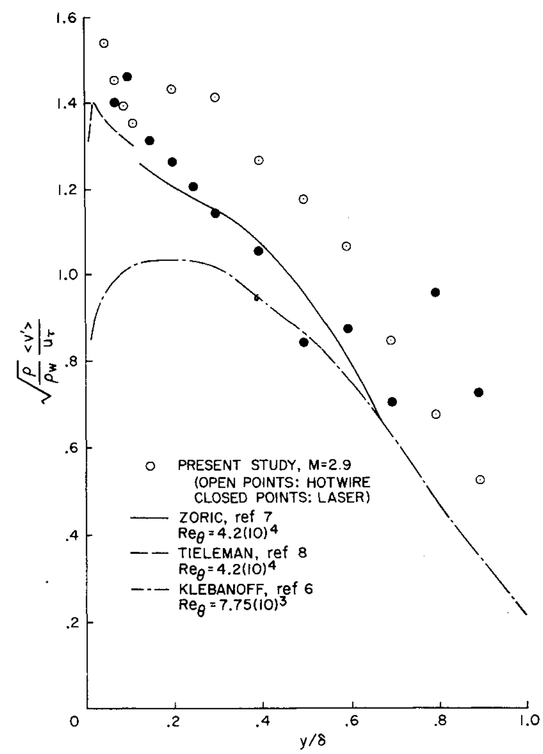

Fig. 2 Cross-stream velocity fluctuation.

high Reynolds number data of Zoric and Tieleman than with the data of Klebanoff.

From the determination of the $\overline{u'v'}$ correlation and knowledge of the local fluid density, the Reynolds shear stress term, $\bar{\rho}\overline{u'v'}$, can be calculated. For a supersonic, boundary-layer-type flow, this term by itself is generally considered to adequately represent the Reynolds shear stress.[5] The $\bar{\rho}\overline{u'v'}$ results of the present study, made dimensionless with respect to τ_w as determined from Preston tube data,[1] are presented in Fig. 3. The shear stress results show some scatter that probably results from the signal differencing required in both techniques. The relative magnitudes of the quantities differenced, however, were not the same for the two instruments since the quantities sensed were different (i.e., just velocity for the laser velocimeter; mass flux and total temperature for the hot-wire anemometer). The hot-wire measurement of $\overline{u'v'}$ could also be affected by pressure fluctuations. However, the general agreement of $\overline{u'v'}$ with the laser velocimeter indicates that pressure fluctuations had no extreme effect on the values.

Figure 3 also includes Klebanoff's[6] incompressible results and the "best estimate" given by Sandborn[10] for the supersonic shear stress distribution. This "best estimate" is based on a compilation of the many supersonic shear stress distributions obtained by integrating mean flow data. In the outer two-thirds of the boundary layer, the present data are generally consistent with the "best estimate." However, as the wall is approached, both the laser velocimeter and hot-wire anemometer data show a fall-off in the shear stress. This is somewhat upsetting since the shear stress is expected to approach the wall value with zero slope under the zero pressure gradient condition. The molecular contribution to the shear stress was negligible even at the measurement station nearest the wall. In Fig. 4, values for the correlation coefficient R_{uv} as determined from the $\langle u' \rangle$, $\langle v' \rangle$, and $\overline{u'v'}$ data are presented. This figure indicates that this quantity, rather than remaining constant as in the incompressible case,[6] also falls off in the inner third of the boundary layer. If, as in the incompressible case,[6] the energy spectra for $\overline{u'v'}$ is similar to that for $\langle u' \rangle$ and $\langle v' \rangle$, the expected effect of inadequate frequency response of the measurement devices would be low estimates for all three quantities $\langle u' \rangle$, $\langle v' \rangle$, and $\overline{u'v'}$, and not just $\overline{u'v'}$ as the decrease in R_{uv} near the wall indicates. The falloff in only $\overline{u'v'}$ suggests a spatial resolution problem with the measurement devices. However, at least for the laser velocimeter, this should not have been a problem since the effective diameter of the sensing volume was only 0.3 mm.

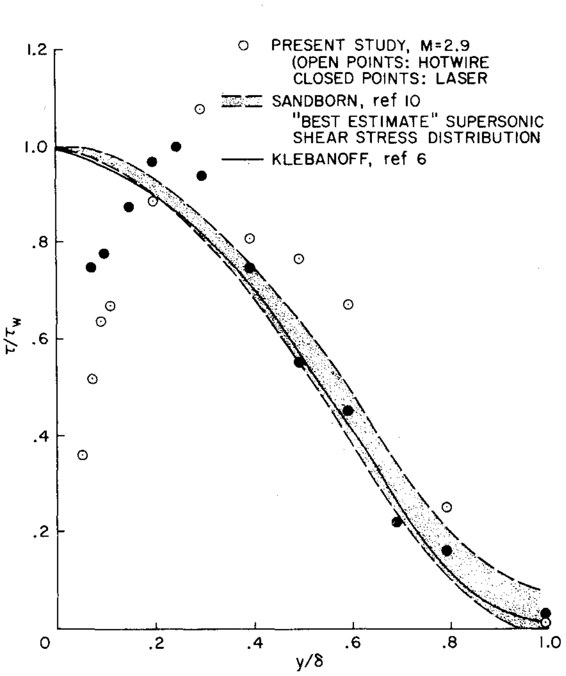

Fig. 3 Reynolds shear stress.

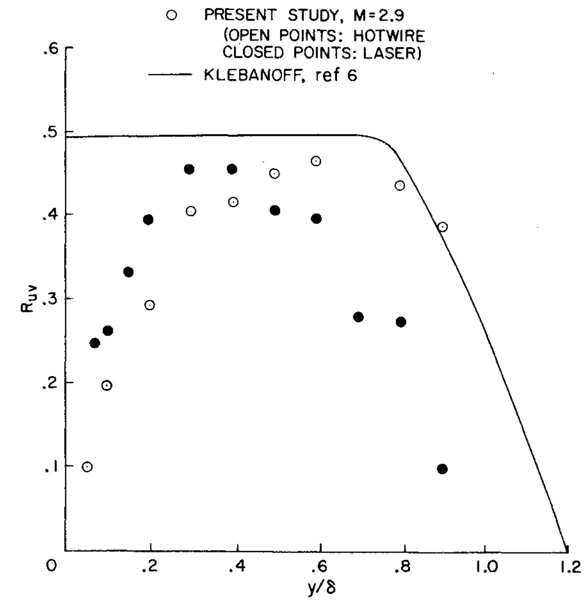

Fig. 4 Correlation coefficient, R_{uv}.

Another possibility is that the data are correct but the shear stress is not adequately represented by $\bar{\rho}\overline{u'v'}$ alone. In the determination of the supersonic shear stress distribution from an integration of mean flow data, the turbulent shear stress is taken to be[10]

$$\overline{(\rho v)'u'} = \bar{\rho}\overline{u'v'} + \bar{v}\overline{\rho' u'} + \overline{\rho' u' v'}$$

Generally, the last two terms on the right-hand size are considered to be negligible compared to $\bar{\rho}\overline{u'v'}$. It is possible that the triple correlation $\overline{\rho' u' v'}$ is not negligible and its contribution accounts for the discrepancy in the shear stress data of this study and the calculated distributions ($\bar{v}\overline{\rho' u'}$ can be discounted as the source of the discrepancy since its sign is opposite that of $\bar{\rho}\overline{u'v'}$).

IV. Conclusions

Turbulence quantities in a supersonic boundary-layer flow have been measured with a laser velocimeter and a hot-wire anemometer. Good agreement between the two techniques for measuring the turbulence quantities, $\langle u' \rangle$, $\langle v' \rangle$, and $\overline{u'v'}$ was obtained. Admittedly, agreement between the two measurement devices in itself is not sufficient to prove the validity of the data. However, considering that agreement between two measurement devices based on completely different physical principles was obtained, and that these data are generally consistent with previous incompressible results, the present study strongly suggests that both measurement techniques can provide reasonably accurate turbulence data in a supersonic boundary layer. Some reservation must be made in regard to the shear stress measurements in the inner third of the boundary layer since the rolloff observed is not consistent with present theory. Further investigation is necessary to establish whether the rolloff in shear stress results from measurement errors or from the assumption that $\bar{\rho}\overline{u'v'}$ adequately represents the compressible turbulent shear stress.

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Aeroelastic Panel Optimization with Aerodynamic Damping

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Nomenclature

a = panel length
D = panel bending stiffness
g = aerodynamic damping parameter, $\rho V a^2(M^2-2)/[\pi^2(m_o D_o)^{1/2}(M^2-1)^{3/2}]$
m = panel mass per unit length
M = freestream Mach number
p = nondimensional slope, w'
q = nondimensional bending moment, $t^3(x)w''$
r = nondimensional shear, $[t^3(x)w'']'$
R_x = inplane loading parameter, $N_x a^2/D_o$; N_x = tensile stress
t = nondimensional thickness ratio, T/T_o; T = panel thickness
t_{min} = minimum allowable thickness ratio
u = unconstrained control function; see Eq. (5)
V = freestream air speed
w = panel deflection divided by panel length a
x = distance along panel divided by panel length a
α^2 = ratio of fundamental flutter frequency to fundamental frequency of free vibration
λ = critical aerodynamic flutter parameter, $\rho V^2 a^3/[D_o(M^2-1)^{1/2}]$
ρ = freestream air density

Subscripts

I = imaginary part
R = real part
o = uniform reference panel value

Introduction and Problem Statement

SEVERAL authors[1–7] have investigated a class of one-dimensional, minimum-weight, supersonic panel design problems in which the flutter speed is specified. Optimal thickness distributions are available for both homogeneous and sandwich panels, and for various levels of constant inplane loading and specified minimum thickness. The purpose of this Note is to present some numerical solutions for the case in which aerodynamic damping is included in the problem formulation.

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Under the assumptions of linear elastic bending and linearized aerodynamic strip theory for high Mach number supersonic flow,[8] the nondimensional equation of equilibrium for assumed simple harmonic motion of an infinite-span solid panel is

$$[t^3(x)w'']''+R_x w''+\lambda_o w'+i\alpha^2\pi^4 g_o w-(\alpha\pi)^4 t(x)w=0 \quad (1)$$

The fourth term of Eq. (1) represents the damping due to aerodynamic forces. Since this term is imaginary, Eq. (1) is complex and may be regarded as two 4th-order dynamic systems coupled by the damping term. For a simply supported solid panel, the problem is to find that thickness ratio distribution $t(x)$, $0 \leqq x \leqq 1$, and fundamental frequency parameter α which minimize the mass ratio $\int_0^1 t(x)\,dx$ subject to the differential equations

$$\begin{aligned} w'_R &= p_R;\quad p'_R = q_R/t^3(x);\quad q'_R = r_R \\ r'_R &= -R_x q_R/t^3(x)-\lambda_o p_R+\alpha^2\pi^4 g_o w_I+(\alpha\pi)^4 t(x)w_R \\ w'_I &= p_I;\quad p'_I = q_I/t^3(x);\quad q'_I = r_I \\ r'_I &= -R_x q_I/t^3(x)-\lambda_o p_I-\alpha^2\pi^4 g_o w_R+(\alpha\pi)^4 t(x)w_I \end{aligned} \quad (2)$$

with boundary conditions

$$\begin{aligned} w_R(0) = q_R(0) = w_I(0) = q_I(0) = 0 \\ r_R(0) = 1 \\ w_R(1) = q_R(1) = w_I(1) = q_I(1) = 0 \end{aligned} \quad (3)$$

and subject to the inequality constraint

$$t(x) \geqq t_{min},\quad 0 < t_{min} < 1,\quad 0 \leqq x \leqq 1 \quad (4)$$

The inplane loading parameter R_x and the aerodynamic damping parameter g_o are regarded as specified constants. The value of the aerodynamic parameter λ_o is held fixed at its critical value for flutter onset which in turn depends on R_x and g_o.[8] Since the dependent variables of Eq. (2) can be arbitrarily scaled without altering the necessary conditions for optimality,[7] an advantageous choice of scaling can be made by setting $r_R(0) = 1$ as shown in Eq. (3).

Plaut[9] has applied a two-term Ritz procedure to the related problem of maximizing the critical aerodynamic parameter λ_o while maintaining a given total panel mass. His simplified analysis, which is apparently the only available result regarding damping for these panel optimization problems, indicates that the addition of damping can have a significant effect. However, it is difficult to predict general trends for the minimum-weight problem from his results. A discussion of the effects of damping on the flutter speed of stressed panels of uniform thickness can be found in Refs. 10 and 11.

Numerical Results

The gradient projection method of Ref. 7 was used to obtain the solutions presented here. The main computational building block in this optimal control method involves the forward integration of the 8th-order system (2) followed by a backward integration of a 32nd-order system of influence function equations used to enforce satisfaction of the four boundary conditions at $x = 1$. The sum of the squares of these four terminal values is required to be less than 10^{-12} at the end of each iteration. The numerical integration is carried out using a 4th-order Runge-Kutta technique with a fixed step size of 0.01. All

Table 1 Critical aerodynamic parameter values[8]

g_o	λ_o
0.00	343.356 (Refs. 4 and 7)
0.50	351
1.00	377
1.25	391
1.50	416
1.75	439
1.80	444